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Inter-sub-band collective excitations in Fibonacci superlattices

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Abstract. In this paper the linear response theory is used to obtain the transfer matrix of the induced potential in a semiconductor superlattice system, in which the sub-band structure is taken into account. Using the transfer matrix to the Fibonacci superlattice, we calculate the spectrum of the intra-sub-band and inter-sub-band collective excitations in such a system. The scaling properties of the spectrum are analysed.

Following the recent experimental discovery of the quasi-crystal phase in metallic alloys [1], there has been a regeneration of interest in studies of the physical properties of quasi-periodic systems in one dimension [2–8]. More recently, Merlin *et al* [9] have succeeded in growing a quasi-periodic semiconductor superlattice, which provides a system for exploring the quasi-periodic properties. This system consists of two building blocks of different thicknesses a and b, arranged in a Fibonacci sequence. Each block is composed of GaAs and GaAlAs layers. If the GaAlAs region is doped with donors, a layer of quasi-two-dimensional electron gas can be produced in every block. Accordingly, the whole system can be thought of as an array of electron gas layers, separated by distances a or b, arranged in a Fibonacci sequence (we assume that the electrons are completely confined in the GaAs potential well). If there are F_m quasi-2D electron gas layers, where F_m is a Fibonacci number, i.e., F_m satisfies the recursion relation $F_{m+1} = F_m + F_{m-1}$, with $F_0 = 1$, $F_1 = 1$, we write the string of the layer separations as S_m, and call it the mth generation of the Fibonacci sequence. This can be constructed recursively as $S_m = \{S_{m-1}, S_{m-2}\}$, with $S_1 = \{a\}$, $S_2 = \{a, b\}$.

Collective excitations in periodic superlattices are well understood [10]. In this system plasmons can propagate along the superlattice direction, and the allowed plasmon frequencies form bands characterised by Bloch wavevectors. However, due to the lack of periodic conditions the Bloch theorem is not applicable to the quasi-periodic system. Das Sarma *et al* [11] studied numerically the plasmon spectrum in a finite quasi-periodic superlattice system. The calculation of the collective excitations for an infinite system has been formulated by Hawrylak and Quinn [12, 13], who supposed that the electrons are confined in the pure 2D layers, i.e., the electron wavefunction is described by a δ-function, with the use of a Poisson equation and suitable boundary conditions. A transfer

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matrix for the induced potential in the system is obtained. However, neither Das Sarma *et al* nor Hawrylak *et al* took the sub-band structure into account, and the inter-sub-band plasmons cannot be given within their frameworks. In this paper, we use the linear response theory for the quasi-periodic superlattice system in which the electron wavefunction has a finite width, and obtain a transfer matrix for the induced potential propagating over the system. Therefore, together with a procedure similar to that of Hawrylak *et al*, not only can the intra-sub-band excitation be obtained more precisely, but the inter-sub-band excitation can also be given.

The model is shown in figure 1, where l is a layer index of the quasi-2D electron gas (2DEG) layer, z_l is the centre of a 2DEG layer along the superlattice direction, and d_l is the distance between the lth and $(l+1)$th layers. For convenience, we assume that the thicknesses of GaAs layers in all blocks are the same, so that the wavefunction for an electron with momentum $\boldsymbol{k}$ (in the x–y plane) in the nth sub-band of the lth layer is

$$|nkl\rangle = \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\Phi_n(z - z_l) \tag{1}$$

and its energy is

$$E_{nkl} = E_n + \hbar^2 k^2/2m^* \tag{2}$$

where E_n is the energy at the bottom of nth sub-band and m^* is the effective electron mass.

An external potential of the form

$$U^{\mathrm{ext}}(\boldsymbol{r}, z, t) = U^{\mathrm{ext}}(\boldsymbol{q}, w, z)\,\mathrm{e}^{\mathrm{i}(\omega t - \boldsymbol{q}\cdot\boldsymbol{r})} \tag{3}$$

induces a change in the electron density, which in turn generates a Hartree potential U^{H} (here we neglect the induced exchange–correlation potential). Then the total perturbation potential

$$U = U^{\mathrm{ext}} + U^{\mathrm{H}} \tag{4}$$

will have the same form as (3). The charge density $\delta n(\boldsymbol{q}, \omega, z)$ induced by the total potential $U(\boldsymbol{q}, \omega, z)$ can be written as

$$\delta n(\boldsymbol{q}, \omega, z) = \sum_{nn'l} \Pi_{nn'}(\boldsymbol{q}, \omega) U^l_{nn'}(\boldsymbol{q}, \omega)\Phi_n(z - z_l)\Phi_{n'}(z - z_l) \tag{5}$$

where

$$U^l_{nn'}(\boldsymbol{q}, \omega) = \int \mathrm{d}z\, \Phi_n(z - z_l)\Phi_{n'}(z - z_l)U(\boldsymbol{q}, \omega, z) \tag{6}$$

and the polarisability Π is

$$\Pi_{nn'}(\boldsymbol{q}, \omega) = 2\sum_k \frac{f(E_{n',k+q}) - f(E_{n,k})}{E_{n',k+q} - E_{n,k} - \hbar\omega}. \tag{7}$$

The induced density $\delta n(\boldsymbol{q}, \omega, z)$ is related to the induced Hartree potential U^{H} by the Poisson equation

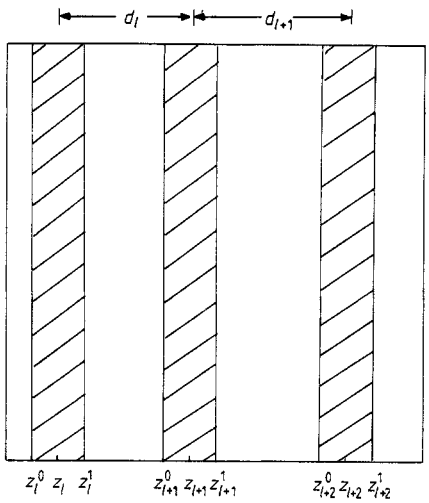

Figure 1. Model of a quasi-periodic semiconductor superlattice. Shaded areas are quantum wells arranged in a Fibonacci sequence, whose centres z_l are separated by distance d_l (a or b). z_l^0 and z_l' are two edges of the lth quantum well.

$$U^{\mathrm{H}}(\boldsymbol{q}, \omega, z) = \frac{2\pi e^2}{\varepsilon q} \int_{-\infty}^{\infty} \mathrm{d}z' \, \mathrm{e}^{-q(z-z')} \delta n(\boldsymbol{q}, \omega, z'). \tag{8}$$

Suppose z is taken to lie in the region with zero electron density ($z_l^1 \leqslant z \leqslant z_{l+1}^0$, see figure 1), from equation (8) we have

$$U^{\mathrm{H}}(\boldsymbol{q}, \omega, z_l^1 \leqslant z \leqslant z_{l+1}^0) = \frac{2\pi e^2}{\varepsilon q} \int_{-\infty}^{z_l^1} \mathrm{e}^{-q(z-z')} \delta n(\boldsymbol{q}, \omega, z') \, \mathrm{d}z'$$
$$+ \frac{2\pi e^2}{\varepsilon q} \int_{z_{l+1}^0}^{\infty} \mathrm{e}^{q(z-z')} \, \delta n(\boldsymbol{q}, \omega, z') \, \mathrm{d}z'. \tag{9}$$

Defining a position parameter z_l^*, which can only take values in the electron layers (i.e., $z_l^0 \leqslant z_l^* \leqslant z_l^1$), equation (9) can be rewritten as

$$U^{\mathrm{H}}(\boldsymbol{q}, \omega, z_l^1 \leqslant z \leqslant z_{l+1}^0) = A_l(z_l^*) \, \mathrm{e}^{-q(z-z_l^*)} + B_l(z_l^*) \, \mathrm{e}^{q(z-z_l^*)} \tag{10}$$

where

$$A_l(z_l^*) = \frac{2\pi e^2}{\varepsilon q} \int_{-\infty}^{z_l^1} \mathrm{d}z' \, \mathrm{e}^{q(z'-z_l^*)} \, \delta n(\boldsymbol{q}, \omega, z') \tag{11a}$$

$$B_l(z_l^*) = \frac{2\pi e^2}{\varepsilon q} \int_{z_{l+1}^0}^{\infty} \mathrm{d}z' \, \mathrm{e}^{-q(z'-z_l^*)} \, \delta n(\boldsymbol{q}, \omega, z'). \tag{11b}$$

Similarly, we can also write

$$U^{\mathrm{H}}(\boldsymbol{q}, \omega, z_{l+1}^1 \leqslant z \leqslant z_{l+2}^0) = A_{l+1}(z_{l+1}^*) \, \mathrm{e}^{-q(z-z_{l+1}^*)} + B_{l+1}(z_{l+1}^*) \, \mathrm{e}^{q(z-z_{l+1}^*)} \tag{12a}$$

where

$$z_{l+1}^* = z_l^* + d_l. \tag{12b}$$

Although $A_l(z_l^*)$ and $B_l(z_l^*)$ are defined in the lth layer, they are not uniquely determined by layer index l because of the appearance of their argument z_l^*. To remove the dependence on z_l^*, we define two new quantities $A_{mn}(l)$ and $B_{mn}(l)$

$$A_{mn}(l) = \int_{z_l^0}^{z_l^1} \mathrm{d}z_l^* \; \Phi_m(z_l^* - z_l)\Phi_n(z_l^* - z_l)A_l(z_l^*) \tag{13a}$$

$$B_{mn}(l) = \int_{z_l^0}^{z_l^1} \mathrm{d}z_l^* \; \Phi_m(z_l^* - z_l)\Phi_n(z_l^* - z_l)B_l(z_l^*). \tag{13b}$$

Since the wavefunction $\Phi_n(z - z_l)$ is non-zero only for $z_l^0 < z < z_l^1$, we can replace z_l^* by z and the integrated area (z_l^0, z_l') by $(-\infty, \infty)$ in the above definitions, i.e.,

$$A_{mn}(l) = \int \mathrm{d}z \; \Phi_m(z - z_l)\Phi_n(z - z_l)A_l(z) \tag{14a}$$

$$B_{mn}(l) = \int \mathrm{d}z \; \Phi_m(z - z_l)\Phi_n(z - z_l)B_l(z). \tag{14b}$$

Using the definitions (11a) and (14a), $A_{mm'}(l+1)$ can be written as

$$A_{mm'}(l+1) = \frac{2\pi e^2}{\varepsilon q}\left(\int_{-\infty}^{z_l^1} \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}z \, \mathrm{e}^{q(z'-z)} \; \Phi_m(z - z_l)\Phi_{m'}(z - z_l)\delta n(\boldsymbol{q}, \omega, z') \, \mathrm{e}^{-qd_l} \right.$$
$$\left. + \int_{z_{l+1}^0}^{z_{l+1}'} \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}z \, \mathrm{e}^{q(z'-z)} \; \Phi_m(z - z_l)\Phi_{m'}(z - z_l)\delta n(\boldsymbol{q}, \omega, z')\right) \tag{15}$$

and from equation (5) we have

$$\delta n(\boldsymbol{q}, \omega, z_{l+1}^0 \leqslant z' \leqslant z_{l+1}') = \sum_{nn'} \Pi_{nn'}(\boldsymbol{q}, \omega)U_{nn'}^{l+1}(\boldsymbol{q}, \omega)\Phi_n(z' - z_{l+1})\Phi_{n'}(z' - z_{l+1}). \tag{16}$$

Inserting (16) into (15), we can obtain

$$A_{mm'}(l-1) = \mathrm{e}^{-qd_l}A_{mm'}(l) + \sum_{nn'} \Pi_{nn'}(\boldsymbol{q}, \omega)U_{nn'}^{l+1}(\boldsymbol{q}, \omega)\tilde{V}_{mm',nn'}(\boldsymbol{q}) \tag{17}$$

where

$$\tilde{V}_{mm',nn'}(\boldsymbol{q}) = \frac{2\pi e^2}{\varepsilon q}\iint \mathrm{d}z \, \mathrm{d}z' \, \mathrm{e}^{q(z-z')}\Phi_m(z)\Phi_{m'}(z)\Phi_n(z')\Phi_{n'}(z'). \tag{18}$$

We also have the similar relation for $B_{mm'}(l+1)$

$$B_{mm'}(l+1) = \mathrm{e}^{qd_l}B_{mm'}(l) - \sum_{nn'} \Pi_{nn'}(\boldsymbol{q}, \omega)U_{nn'}^{l+1}(\boldsymbol{q}, \omega)\tilde{V}_{nn',mm'}(\boldsymbol{q}). \tag{19}$$

On the other hand, when z lies in the electron layers, we can write

$$U^{\mathrm{H}}(\boldsymbol{q}, \omega, z_l^0 \leqslant z \leqslant z_l') = A_l(z) + B_l(z) + \frac{2\pi e^2}{\varepsilon q}\int_{z_l^0}^{z_l'} \mathrm{d}z'(\mathrm{e}^{-q|z-z'|}$$
$$- \mathrm{e}^{-q(z-z')})\delta n(\boldsymbol{q}, \omega, z'). \tag{20}$$

On using (16), multiplying both sides of equation (20) by $\Phi_m(z - z_l)\Phi_{m'}(z - z_l)$, and integrating for z, equation (20) becomes

$$U^l_{mm'}(\boldsymbol{q}, \omega) = U^{\text{ext},l}_{mm'}(\boldsymbol{q}, \omega) + A_{mm'}(l) + B_{mm'}(l) + \sum_{nn'} \Pi_{nn'}(\boldsymbol{q}, \omega) U^l_{nn'}(\boldsymbol{q}, \omega)(V_{mm',nn'}(\boldsymbol{q}) - \tilde{V}_{mm',nn'}(\boldsymbol{q})) \tag{21}$$

where

$$V_{mm',nn'}(\boldsymbol{q}) = \frac{2\pi e^2}{\varepsilon q} \iint \mathrm{d}z\, \mathrm{d}z'\, \mathrm{e}^{-q|z-z'|} \Phi_m(z)\Phi_{m'}(z)\Phi_n(z')\Phi_{n'}(z'). \tag{22}$$

If we put $U^{\text{ext},l}_{mm'}(\boldsymbol{q}, \omega) = 0$, then the set of equations (17), (19) and (21) determines the collective excitation of the system. In the electric quantum limit (i.e., only the lowest sub-band is occupied by electrons), the set of the equations can be simplified as follows

$$A_{m0}(l+1) = \mathrm{e}^{-qd_l} A_{m0}(l) + \sum_n \chi_{n0}(\boldsymbol{q}, \omega) U^{l+1}_{n0}(\boldsymbol{q}, \omega) \tilde{V}_{mn}(\boldsymbol{q}) \tag{23a}$$

$$B_{m0}(l+1) = \mathrm{e}^{qd_l} B_{m0}(l) - \sum_n \chi_{n0}(\boldsymbol{q}, \omega) U^{l+1}_{n0}(\boldsymbol{q}, \omega) \tilde{V}_{mn}(\boldsymbol{q}) \tag{23b}$$

$$U^l_{m0}(\boldsymbol{q}, \omega) = A_{m0}(l) + B_{m0}(l) + \sum_n \chi_{n0}(\boldsymbol{q}, \omega) U^l_{n0}(\boldsymbol{q}, \omega)[V_{mn}(\boldsymbol{q}) - \tilde{V}_{mn}(\boldsymbol{q})] \tag{23c}$$

where

$$\chi_{00}(\boldsymbol{q}, \omega) = \Pi_{00}(\boldsymbol{q}, \omega) \tag{24a}$$

$$\chi_{m0}(\boldsymbol{q}, \omega) = \Pi_{m0}(\boldsymbol{q}, \omega) + \Pi_{0m}(\boldsymbol{q}, \omega) \qquad \text{for } m \neq 0 \tag{24b}$$

$$V_{mn}(\boldsymbol{q}) = V_{m0,n0}(\boldsymbol{q}) \tag{24c}$$

$$\tilde{V}_{mn}(\boldsymbol{q}) = \tilde{V}_{m0,n0}(\boldsymbol{q}). \tag{24d}$$

The above set of equations is general no matter whether the system is periodic or not. In fact, for the periodic superlattice, we can write

$$\begin{pmatrix} A_{mn}(l+1) \\ B_{mn}(l+1) \end{pmatrix} = \mathrm{e}^{\mathrm{i}kd} \begin{pmatrix} A_{mn}(l) \\ B_{mn}(l) \end{pmatrix}$$

using Bloch's theorem and combining it with equations (23) to yield

$$U^l_{m0}(\boldsymbol{q}, \omega) = \sum_n \chi_{n0}(\boldsymbol{q}, \omega)[V_{mn}(\boldsymbol{q}) + S_- \tilde{V}_{mn}(\boldsymbol{q}) + S_+ \tilde{V}_{nm}(\boldsymbol{q})] U^l_{n0}(\boldsymbol{q}, \omega) \tag{25a}$$

$$S_\pm(\boldsymbol{q}, k) = (\mathrm{e}^{qd \mp \mathrm{i}kd} - 1)^{-1} \tag{25b}$$

The equation for determining the excitations can be written as

$$\det\{\delta_{nm} - \chi_{m0}(\boldsymbol{q}, \omega)[V_{nm}(\boldsymbol{q}) + S_- \tilde{V}_{nm}(\boldsymbol{q}) + S_+ \tilde{V}_{mn}(\boldsymbol{q})]\} = 0. \tag{26}$$

This is just the result obtained by Tselis and Quinn [10].

For the quasi-periodic superlattice, due to the lack of the periodic condition, the connection between $(A_{m0}(l+1), B_{m0}(l+1))$ and $(A_{m0}(l), B_{m0}(l))$ can only be written via a transfer matrix $\mathbf{T}_l$

$$\begin{pmatrix} A_{m0}(l+1) \\ B_{m0}(l+1) \end{pmatrix} = \mathbf{T}_l \begin{pmatrix} A_{m0}(l) \\ B_{m0}(l) \end{pmatrix} \tag{27}$$

$$\mathbf{T}_l = \begin{pmatrix} e^{-qd_l}\left(1 + \dfrac{\chi_{m0}\tilde{V}_{mm}}{1-\chi_{m0}(V_{mm}-\tilde{V}_{mm})}\right) & e^{qd_l}\dfrac{\chi_{m0}\tilde{V}_{mm}}{1-\chi_{m0}(V_{mm}-\tilde{V}_{mm})} \\ -e^{-qd_l}\dfrac{\chi_{m0}\tilde{V}_{mm}}{1-\chi_{m0}(V_{mm}-\tilde{V}_{mm})} & e^{qd_l}\left(1 - \dfrac{\chi_{m0}\tilde{V}_{mm}}{1-\chi_{m0}(V_{mm}-\tilde{V}_{mm})}\right) \end{pmatrix} \tag{28}$$

where the diagonal approximation (i.e., neglecting the mixing between different excitations) is made. If setting $n = 0$ and $V_{00} = \tilde{V}_{00} = 2\pi e^2/\varepsilon q$, it is expected that (28) will reduce to the form of a transfer matrix for the pure 2DEG layer system obtained by Hawrylak *et al* [12].

In equation (28) the quantity $\chi_{m0}\tilde{V}_{mm}/[1-\chi_{m0}(V_{mm}-\tilde{V}_{mm})]$, is defined in a quasi-2DEG layer and is independent of layer separations, and therefore of the superlattice structure. Since the allowed energy values are determined by the condition that

$$\prod_{l=0}^{N} T_l$$

which only depends on the superlattice structure, remains finite when $N \to \pm\infty$ (see below), the allowed values of $\chi_{m0}\tilde{V}_{mm}/[1-\chi_{m0}(V_{mm}-\tilde{V}_{mm})]$ are completely determined by the superlattice structure for a fixed wavevector q. If we define a quantity $S(q)$ as

$$S(q) = \frac{1-\chi_{m0}(V_{mm}-\tilde{V}_{mm})}{\chi_{m0}\tilde{V}_{mm}} \tag{29}$$

which is related to $\mathbf{T}_l$ by

$$\mathbf{T}_l = \begin{pmatrix} e^{-qd_l}[1+S^{-1}(q)] & e^{qd_l}S^{-1}(q) \\ -e^{-qd_l}S^{-1}(q) & e^{qd_l}[1-S^{-1}(q)] \end{pmatrix} \tag{30}$$

and write its allowed values as $S(q, k)$, where k is an index specifying the allowed values of $S(q)$, then the equation of the collective excitations can be given by

$$\chi_{m0}(q,\omega)\tilde{V}_{mm}(q)S(q,k) = 1 - \chi_{m0}(q,\omega)[V_{mm}(q) - \tilde{V}_{mm}(q)]. \tag{31}$$

The above procedures are performed on general systems, but if we recall the definition of the structure factor and the equation of collective excitations in the periodic superlattice, we find that (29) is just the definition of a structure factor for the periodic system, and that equation (31) is also of the same form as that of the collective excitations in that system. The only difference lies in the allowed values of $S(q)$, i.e., $S(q, k)$, which can be determined by different conditions for the different systems. Therefore, $S(q)$ can be regarded as the structure factor for an arbitrary system.

Although the discussions are based on the random-phase approximation (RPA), they hold for any approximations provided that the effect of tunnelling between layers is negligible. This means that, for any approximation, there always exists a quantity (we call it a structure factor, $S(q)$) of the 2DEG layer, which may be different for the different

approximations but the same for the different superlattice structures. Its allowed values $S(q, k)$ are uniquely determined by the superlattice structure. Thus the equations of the collective excitations, such as equation (31) in the RPA treatment, are the same for the different superlattice systems. The only difference lies in $S(q, k)$ which is determined by the corresponding equations for the different superlattice structures. Physically, this is understandable since the tunnelling effect is neglected, and the single-electron states and related quantities are not influenced by the superlattice structure, i.e., are the same for all systems.

The polarisability $\Pi_{nn'}(q, \omega)$ is just one of these quantities, and its allowed values are determined by the superlattice structure, from which the energy spectrum can be obtained. Since the structure factor is only the combination of $\Pi_{nn'}(q, \omega)$ and some other 2DEG layer quantities, the above conclusion is physically obvious.

For the quasi-periodic system, the string of $\mathbf{T}_l$ is arranged in a Fibonacci sequence. The allowed values of $S(q)$ in equation (30) can be studied by the same method as that used by Kohmoto *et al* [3], Ostlund and Pandit [7] and Hawrylak and Quinn [12], i.e., by the rational approximation method. An mth rational approximation to a Fibonacci sequence consists of a periodic sequence of unit cells containing F_m matrices $\mathbf{T}$ obtained in the mth generation of a Fibonacci sequence. The allowed values of $S(q)$ in mth rational approximation can therefore be determined by the equation

$$\tfrac{1}{2}\left|\operatorname{Tr}\prod_{l=1}^{F_m}\mathbf{T}_l\right| \leqslant 1. \tag{32}$$

As shown in [3], the quantity

$$\chi_m = \tfrac{1}{2}\operatorname{Tr}\prod_{l=1}^{F_m}\mathbf{T}_l$$

satisfies the recursion relation

$$\chi_{l+1} = 2\chi_l\chi_{l-1} - \chi_{l-2}. \tag{33}$$

The starting conditions are

$$\chi_1 = \tfrac{1}{2}\operatorname{Tr}(\mathbf{T}_a) = \cosh(qa) - S^{-1}(q)\sinh(qa) \tag{34a}$$

$$\chi_0 = \tfrac{1}{2}\operatorname{Tr}(\mathbf{T}_b) = \cosh(qb) - S^{-1}(q)\sinh(qb) \tag{34b}$$

and χ_{-1} should be chosen as

$$\chi_{-1} = \cosh[q(a-b)] \tag{34c}$$

to give $\chi_2 = \frac{1}{2}\operatorname{Tr}(\mathbf{T}_b\mathbf{T}_a)$ from equation (33) with χ_1 and χ_0 as defined above. Then, from (32) we can obtain the allowed values for any rational approximations.

In the long-wavelength approximation, we expand $\chi_{m0}(q, \omega)$ to $O(q^2)$. If the allowed values of the structure factor $S(q)$ are obtained, equation (31) yields the intra-sub-band excitation energy

$$\omega^2 = \frac{nq^2}{m^*}V_0(q) \tag{35}$$

and the inter-sub-band excitation energy

$$\omega^2 = \omega_{m0}^2\left(1 + \frac{2n_s}{\hbar\omega_{m0}}V_m\right) + \left[\frac{\hbar\omega_{m0}}{m^*}\left(1 + \frac{2n_s}{\hbar\omega_{m0}}\frac{V_m}{2}\right) + v_F^2\left(\frac{3}{4} + \frac{\hbar\omega_{m0}}{2n_s}\frac{1}{V_m}\right)\right]q^2 \tag{36}$$

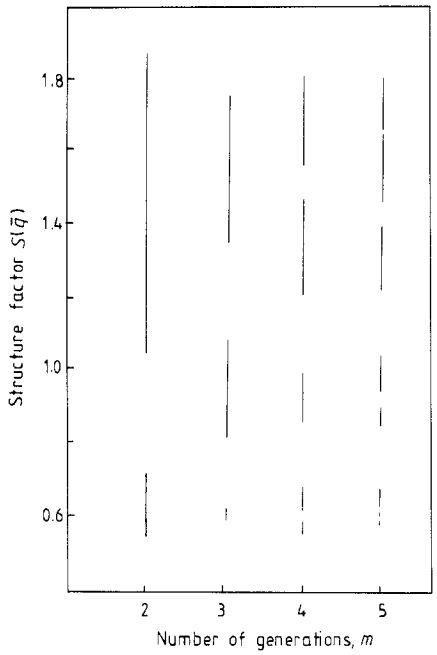

Figure 2. Structure factor $S(q)$ for $qb = 1$. The band structures for $m = 2, 3, 4, 5$ are shown, where m is the number of generations in the Fibonacci sequence.

where

$$V_m(q) = V_{mm}(q) + [S(q) - 1]\tilde{V}_{mm}(q) \tag{37}$$

and n_s is the electron density, $\omega_{m0} = (E_m - E_0)/\hbar$, v_F is the Fermi velocity.

In performing the numerical calculation, we use the wavefunction and single-particle energy corresponding to a quantum well with an infinite potential barrier, that is,

$$\Phi_m(z) = \sqrt{2/L}\sin[(m+1)\pi(z/L + \tfrac{1}{2})] \qquad E_m = (\hbar^2/2mL^2)(m+1)^2\pi^2$$

with the following parameters: $\varepsilon = 13.1$, $n_s = 7.3 \times 10^{11}\ \mathrm{cm}^{-2}$, $m^* = 0.068m_0$ (m_0 is the free-electron mass), $L = 250$ Å, $b = 500$ Å and $a = b\tau$ (τ is the golden mean $\frac{1}{2}(1 + \sqrt{5})$). For $qb = 1$, the bands of the quasi-periodic structure factor, the intra-sub-band excitation and the inter-sub-band excitation for various rational approximations in the Fibonacci sequence are plotted in figures 2, 3 and 4, respectively.

The scaling of the spectrum can be studied using the map (equation (33)) and the starting condition (34). Kohmoto and Oono [5] have discussed the scaling properties of the map (33) using the fixed-point analysis for the quasi-periodic potential Schrödinger equation. They rewrite the map (33) as a three-dimensional map M

$$M(x_l, y_l, z_l) = (x_{l+1}, y_{l+1}, z_{l+1}) = (2x_l y_l - z_l, x_l, y_l) \tag{38}$$

with

$$x_{l+1} = \chi_{l+1} \qquad y_{l+1} = \chi_l \qquad z_{l+1} = \chi_{l-1} \tag{39}$$

and find an important conserved quantity λ^2,

$$\lambda^2 = x_l^2 + y_l^2 + z_l^2 - 2x_l y_l z_l - 1 \tag{40}$$

which is uniquely determined by the quasi-periodic potential and is independent of the

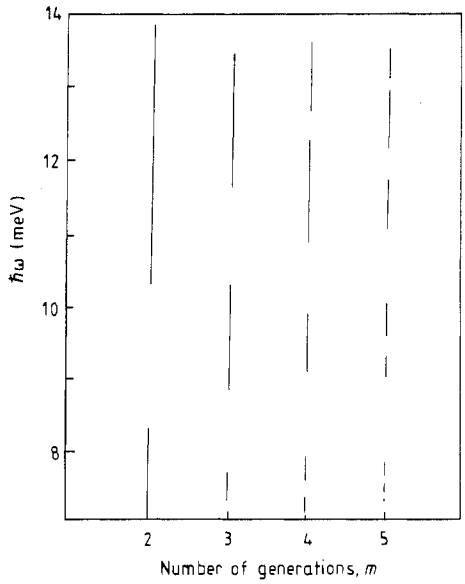

Figure 3. The band structure of intra-sub-band excitation for $m = 2, 3, 4, 5$.

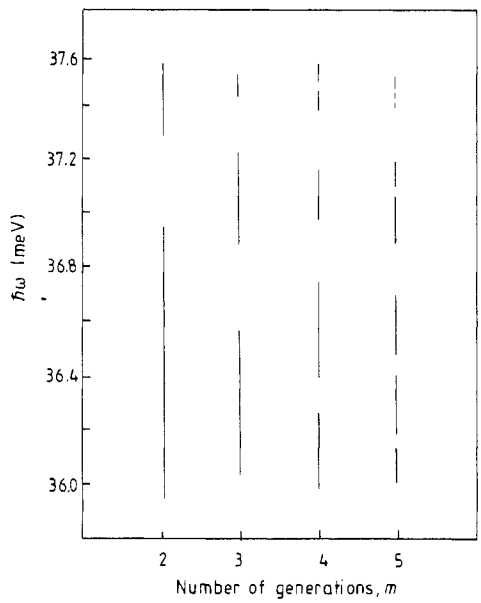

Figure 4. The band structure of inter-sub-band excitation for $m = 2, 3, 4, 5$.

energy in their discussions. Kohmoto and Oono also find six fixed points of M^6. With the existence of the conserved quantity λ^2, the points obtained by successive iterations (38) are confined on a two-dimensional surface (manifold), which is uniquely determined by λ^2. For a given system, λ^2 is definite and so is the manifold, so the starting points in their system constitute a line on the manifold naturally; and the points on the line near a fixed point are responsible for the scaling of the energy spectrum.

However, the case is different for the quasi-periodic superlattice, in which λ^2 is a function of energy (or the structure factor) and wavevector; the starting points constitute a two-dimensional surface. Only those points on the surface with energies and wavevectors satisfying relation (40) are on the manifold determined by the same equation, and form a crossed line of the surface and the manifold. Since two arbitrary points on the line have different wavevectors, then, strictly speaking, the conclusions of Kohmoto *et al* do not hold for the spectrum of the Fibonacci superlattice, which is calculated with a definite wavevector. However, only those points (we call them stable points) on the crossed line that are near to a fixed point are interesting in a discussion of the scaling of the spectrum. Hence we can treat the wavevectors of the points to be approximately the same q_0 value, and the qualitative conclusions then hold for the spectrum of our system with wavevector q_0. Furthermore, since the wavevectors of the stable points are determined by the given conserved quantity λ^2, which we can take to be arbitrary, the scaling properties studied by Kohmoto *et al* hold approximately for the energy spectrum of our system with an arbitrary wavevector. This means that the band structure of the spectrum exhibits an infinite number of very narrow bands as $m \to \infty$, which have a self-similar Cantor set structure with the scaling. More detailed discussions of the scaling of the specrum will be given in the future.

In conclusion, we have given a transfer matrix theory for studying the collective excitations in a quasi-periodic superlattice system, and obtained the energy spectrum of the intra-sub-band and the inter-sub-band charge density excitations. Experimentally, as proposed by Das Sarma *et al* [11] and Hawrylak and Quinn [12, 13], these spectra can be detected by inelastic light scattering. We hope the present work will stimulate experiments to investigate the energy spectrum.

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